

# Chaotic Analysis of Electroencephalographic Signal for Sleep Quality Measurement

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**Abstract:** Determination of the sleep stages is essential for sleep quality measurement, which is associated with person's daily activity, emotional disturbance, accidental risk, and illness. In this paper, we analyze electroencephalographic (EEG) signals by means of chaotic analysis methods. The EEG signals measured during sleep were modeled as the deterministic chaos system, whose representation was performed by phase space transformation. The extracted parameters from EEG signals include the delay time, the embedded dimension, the correlation dimension and the largest Lyapunov exponent. Throughout experimental observations, it has been demonstrated that extracted parameters can serve as a relative index for sleep stages identification. The difference between the light sleep and deep sleep was well discernible by all the extracted parameters, but differences within light sleep (between light sleep 1 and light sleep 2) and within deep sleep (between deep sleep 1 and deep sleep 2) were not significant. The standard error of the mean is helpful to discriminate wakefulness, and rapid eye movement (REM), in spite of their similar EEG signal patterns.

**Keywords:** *chaos, sleep stage determination, sleep quality*

## 1. INTRODUCTION

Sleep quality is associated with daily activity and the illness in human. Sleep disturbance is relevant to many disorders, including sleep apnea, insomnia, depression, schizophrenia, and Alzheimer disease [7]. In addition to illness, sleep deprivation can disturb the emotional behavior [7]. Insufficient sleep can result in the sleepiness, causing troubles in concentration and attention during task fulfillment as well as increasing the risk of accidents [4]. Hence, the assessment of sleep quality is essential for the maintenance of healthy life, the enhancement of daily performance, the prevention of accident, and the diagnosis and treatment of sleep associated disorders.

In hospital, sleep quality is clinically assessed by polysomnography, which measures bio signals including EEG, electrooculogram (EOG), and electromyogram. By visually analyzing measured bio signals, sleep stages are identified, which are the basis of sleep quality assessment. The duration and cyclic pattern of sleep stages are related with the sleep quality. Sleep stages are classified into two categories; REM sleep and non-REM sleep, which are generally repeated 4 to 5 times during the night. Non-REM sleep is composed of light sleep and deep sleep. In accordance with sleep patterns observed during the whole sleep period, light sleep and deep sleep can be further divided into light sleep 1 and light sleep 2,

and deep sleep 1 and deep sleep 2, respectively.

Besides manual inspection of measured bio signals, the computerized analysis of sleep stages based on bio-signals can provide the efficient means to evaluate the sleep quality [1]. Among many bio signals, EEG is one of the most prominent bio signals for the determination of sleep quality [5]. Diverse processing methods for EEG data, including linear time series analysis and spectral analysis, have been expanded into non-linear dynamical system analysis. Generally, the brain can be modeled as complex feedback control system connecting activated neurons dynamically from different parts of brain, which can be suitable for non-linear analysis. Although it is not clear whether different sleep stages truly reflect the non-linear dynamics or not [6], non-linear analysis method can provide additional cues in quantifying sleep quality. Because of the complexity and the limited predictability of the EEG, the deterministic chaos can be applied to nonlinear EEG analysis. The rapid loss of predictability or temporal unpredictability of the EEG signal patterns can be the characteristics of the dissipative chaotic system [13].

In this paper, we applied methods of the non-linear dynamical systems associated with deterministic chaos system, to measured EEG signals during sleep. They can be used in the study of self-organization and pattern formation in the complex neural networks of the brain occurred differently during each sleep stages.

The chaotic parameters, including the delay time, the embedded dimension, the correlation dimension (D2) and the Lyapunov exponent (L1), were applied to EEG signals to quantify the complexity of the physiological phenomenon, that is reflected on the sleep EEG signals associated with different sleep stages.

## 2. MATERIALS AND METHODS

### 2.1 Experiment

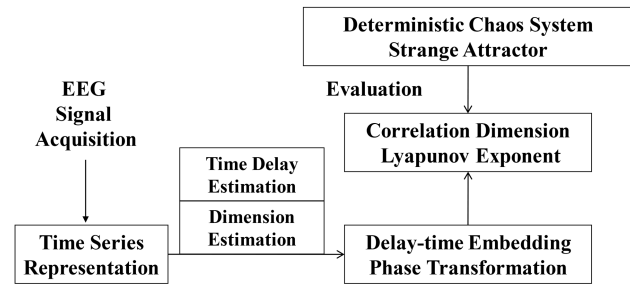
Four healthy young men between the ages of 27 and 29 years, whose mean age is 27.5 years, participated in the present study. They did not use any medications and had no sleep complaints. The participants were asked to go to bed between 10 and 12 pm and were permitted to sleep for a maximum of 8 hours. All recording were preceded by at least one adaptation night in the sleep laboratory to accommodate sleep experimental environment.

Poly-graphic recordings (BIOPAC MP150 system) including EEG, EOG and EMG were obtained. The chin EMG was recorded at the submental region, and the EOG electrodes were placed on the outer cantus of the left and right eyes [4,5]. EEG electrodes were placed at C3 and C4. Monopolar recording with ear-reference was used for EEG acquisition. EEG was measured epoch-by-epoch in 30 seconds interval with a sampling rate of 1 kHz, a gain of 10,000, high pass cutoff frequency of 0.5Hz, low pass cutoff frequency of 100Hz, and the 60 Hz notch filter.

The measured EEG signals are scored by manual inspection using poly-graphic recordings (EEG, EOG, and EMG) according to “a manual of standardized terminology, techniques and scoring system for sleep stages of human subjects”, called R & K rule [3]. Sleep stages were classified into six stages for later chaotic analysis; wakefulness (WF), light sleep stage 1 (LS1), light sleep stage 2 (LS2), deep sleep stage 1 (DS1), deep sleep stage 2 (DS2), and rapid eye movement (REM).

### 2.2 Methods

As shown in Figure 1, the measured EEG signals were modeled by the deterministic chaos system (strange attractor) [6]. The EEG signals during sleep have an apparently noisy behavior (for example sleep spindles and k-complexes) but are ruled by deterministic laws (for example delta, theta and alpha oscillations) [12]. The chaos analysis starts from the time series representation of the measured EEG signal of size  $N$ . Then, it is common to transform the evolution of the time series EEG data  $\{s(1), s(2)\dots\}$  into the phase space representation



**Figure 1:** The flow chart to process the EEG signals by chaos analysis method

$\{x(1), x(2) \dots\}$ . The phase space transformation can be performed by delay-time embedding, referred to as the Taken's reconstruction [13]. An  $m$ -dimensional phase space is spanned by a set of  $m$ -dimensional embedding vectors. Each embedding vector defines a point in a phase space, representing the instantaneous state of the system.

$$x(1) = s(1), s(1+L), s(1+2L), \dots, s(1+[m-1]L)$$

$$x(2) = s(2), s(2+L), s(2+2L), \dots, s(2+[m-1]L)$$

$$x(k) = s(k), s(k+L), s(k+2L), \dots, s(k+[m-1]L)$$

The  $m$ ,  $L$  and  $k$  represent the embedded dimension, the time delay, and the number of  $m$ -dimensional vectors, respectively. For phase space transformation,  $N$  and  $k$  should be carefully chosen and  $L$  and  $m$  should be estimated.

Mutual information method is used to estimate the time delay ( $L$ ). The time delay is determined as the zero crossing point where the first derivative of the mutual information function [2] passes through. If estimated  $L$  is too small, then each embedded vectors are located so closely (redundancy). If  $L$  is too large, then each embedded vectors are independent (irrelevance).

False nearest neighbors method proposed by Kennel [8] is used to estimate embedded dimension,  $m$ . The algorithm is based on the idea that in the passage from the dimension  $m$  to  $m+1$ , one can differentiate between point on the orbit that are true and false neighbors. The nearest neighbor embedding vectors in the phase space still remain close for the increased dimension  $m+1$ . If not close, increase the  $m$ . That is, the attractor is not completely unfolded, and the embedded dimension must be higher. The increase of  $m$  is repeated until neighbors remain close.

The correlation dimension measures the complexity of the system related with the dimension, as well as can discriminate between the deterministic and random systems. Grassberger-Procaccia algorithm (GPA) [10] is used to compute the correlation dimension. GPA is based

on the correlation sum,  $C(r)$ , which is defined as the average number of points on the reconstructed attractor within radius  $r$ . Each vector defines a point in the  $m$ -dimensional embedding space.

$$C(r) = \lim_{N \rightarrow \infty} \frac{1}{N^2} \sum_{i \neq j} \Theta(r - |\mathbf{x}(i) - \mathbf{x}(j)|)$$

$N$  is the number of vectors, and  $\Theta$  is the Heaviside function. The correlation dimension is defined as:

$$D_2 = \lim_{r \rightarrow 0} \frac{\log C(r)}{\log r}$$

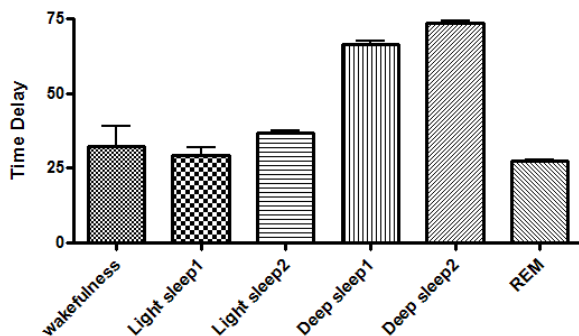
Lyapunov exponent quantify the exponential sensitivity of the divergence from the initial condition and the level of the chaos of a system. Rosenstein algorithm [9] is used to measures the exponential divergence of the trajectory in the phase space. As more rapidly two trajectories diverge for a certain period of time, the more chaotic is the system (the more sensitive to initial condition).

$$d(t) \rightarrow ae^{\lambda t}$$

The  $a$  and  $d(t)$  are the beginning distance and distance at time  $t$  between two close trajectories in the phase space, respectively. The  $d(t)$  is proportional to  $a$  at  $\lambda$  and  $\lambda$  is the largest Lyapunov exponent. The longest axis corresponding to the most unstable direction can be determined by the largest Lyapunov exponent. Positive value of  $\lambda$  implies the presence of chaos [12, 13].

### 3. RESULTS

Figure 2 shows the estimated time delay with respect to different sleep stages (WF, LS1, LS2, DS1, DS2, and REM). The error bar represents the standard error of the mean (SEM), which is defined as the normalized standard deviation (division of the standard deviation by squared

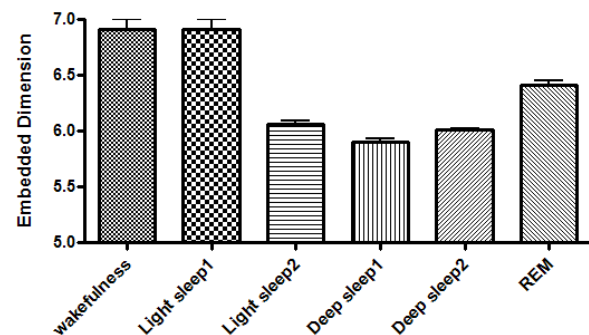


**Figure 2:** Estimated time delay with respect to different sleep stages: WF, LS1, LS2, DS1, DS2, and REM

root of the sample size). The deeper the sleep, the longer the value of time delay. That is, as the sleep gets deeper, EEG signals become more deterministic, less random and less complex. The difference of the delay time between the light sleep and deep sleep is significant, but the mean values of the time delay for WF, LS1, LS2, and REM are not significantly different. However, the SEM for the WF is larger than that for REM, which can indicate that the SEM is plausible index to discriminate between the WF and REM. The former is in wake state, and the latter is in dreaming state. Even though the EEG signals for WF and the REM show the similar physiological patterns, the time delay for a wake state shows a larger variability than that for a sleep state. Similarly, the variability of the SEM for LS1 is larger than those for DS1 and DS2. Thus, the time delay is more variable in wake state than in sleep state. As rule of thumb, the deeper the sleep, the more stable (the less complex) EEG signals.

Figure 3 shows the estimated embedded dimensions with respect to different sleep stages (WF, LS1, LS2, DS1, DS2, and REM). In comparison with the delay time (Figure 2), the characteristics of the embedded dimension changes to opposite. The deeper the sleep, the smaller the mean value of embedded dimension. As the sleep gets deeper, the embedded dimension gets smaller, indicating less complex system. SEM patterns for the embedded dimension are similar comparing with those for the delay time. However, there is a small SEM difference between WF and REM.

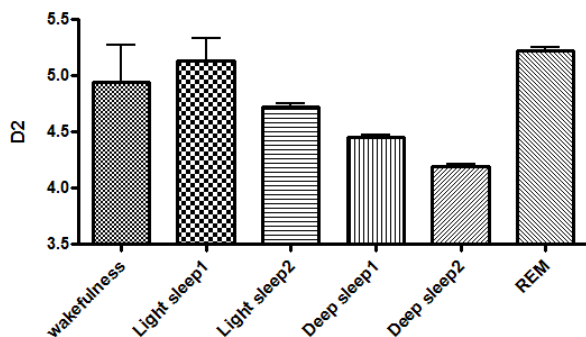
Figure 4 shows the estimated correlation dimension with respect to different sleep stages (WF, LS1, LS2, DS1, DS2, and REM). The characteristics of correlation dimension shows the similar pattern comparing with the embedded dimension (Figure 3). The deeper the sleep, the smaller the mean value of the correlation dimension. As the sleep gets deeper, the embedded dimension gets smaller, indicating less complex system, since the corre-



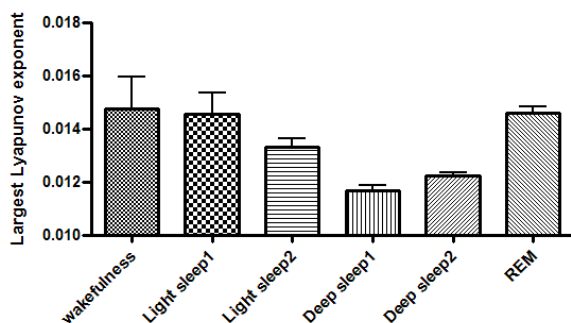
**Figure 3:** Estimated embedded dimensions with respect to different sleep stages: WF, LS1, LS2, DS1, DS2, and REM

lation dimension measures complexity of the system. SEM patterns for the correlation dimension are similar comparing with those for the time delay. Comparing WF with LS1, the mean value of the correlation dimension for WF is smaller than that for LS1, but the SEM for the correlation dimension is larger than that for LS1. The WF is less complex than LS 1 from the point of mean value for the correlation dimension, and the WF is less stationary than LS1 from the point of the SEM for the correlation dimension. The non-integer values of correlation dimension for all the cases (WF, LS1, LS2, DS1, DS2, and REM) represent the property of a strange attractor [12, 13].

Figure 5 shows the estimated largest Lyapunov exponent with respect to different sleep stages (WF, LS1, LS2, DS1, DS2, and REM). The characteristics of the largest Lyapunov exponent shows similar pattern comparing with the correlation dimension (Figure 4). The more light the sleep, the higher the mean value of the largest Lyapunov exponent. As the sleep gets awake, the EEG signal becomes more chaotic, and divergent. Because the largest Lyapunov exponents for all cases are positive, all sleep stages can be represented by chaos system [12, 13]. SEM patterns for largest Lyapunov exponent are similar comparing with those for the correlation dimension.



**Figure 4:** Estimated correlation dimension ( $D_2$ ) with respect to different sleep stages: WF, LS1, LS2, DS1, DS2, and REM



**Figure 5:** Estimated largest Lyapunov exponent with respect to different sleep stages: WF, LS1, LS2, DS1, DS2, and REM

#### 4. DISCUSSION

It has been generally regarded as the sleep is an ingredient for the human life. Sleep is an essential state of rest and refreshment, as well as is closely related with illness. Sleepiness can cause the trouble in concentration and increase the risk of accidents. The sleep quality can be better qualified by identifying each sleep stages and sleep cycles. Rechtschaffen and Kales [3] proposed a method of sleep classification based on visual inspection of bio signals waveform patterns including EEG, EOG, and EMG. That method has been the standard for sleep stage classification for more than 35 years. However, the use of this method for sleep classification is tedious, and highly dependent on the observer's experience. The reliable determination of each sleep stages is difficult even for the expert, because of the ambiguity, uncertainty, and fuzziness between stages. Hence, computer assisted determination of sleep stages could be helpful to gain insight into sleep quality assessment system.

Among many bio signals, EEG is dominant to identify sleep stages. Many types of linear analysis methods in both time and frequency domains have been applied to the EEG signals, but the performances of those methods are sometimes unsatisfactory. EEG sometimes shows regular patterns, but sometimes shows irregular and complex patterns. EEG is similar to a chaotic time series which rapidly becomes unpredictable, and shows random behavior. Because EEG shows chaotic behavior, linear analysis methods are often unsuccessful [4]. EEG signals can be characterized by sensitivity to initial conditions and degree of complexity and chaotic. EEG looks noisy, but is different from the noise. EEG signal may be thought of as deterministic chaos, since brain activity shows an apparently noisy behavior, but is governed by deterministic rules [13]. The deterministic chaos system can be characterized by chaotic (complexity) and dependency to initial conditions. More specifically, EEG signal can be viewed as strange attractor. In strange attractor, the trajectories never exactly repeat again. The trajectories of dissipative chaotic systems manifest spatial structure confine to a finite region [13]. Small difference of EEG pattern for the current state of the brain results in an exponential discrepancy of EEG pattern for later behavior. Additionally, a strange attractor can be reconstructed from a sampled time waveform of just one component of the state, corresponding to the finite data length of  $N$ , which can simplify the EEG measurement procedure (the reduced

total length of measured EEG data) and reduce the storage cost [13].

Chaos analysis starts from transforming a time-series data into a state space representation by phase space transformation [8,12,13]. The chaotic system like EEG can be suitably represented by phase space transformation, where the reconstructed point in the phase space indicates the instantaneous state of the system. The parameters in phase space transformation can include the delay time, the embedded dimension, the correlation dimension, and the largest Lyapunov exponent. Those parameters can serve as a relative index of the complexity of the EEG dynamics. The delay and the dimension are related with the complexity, while exponent is related with the divergence property of chaotic system. Particularly, the correlation dimension can converge to the finite value for deterministic signal [12], but it can diverge for random signal. Low dimensionality and long delay correspond to EEG synchronization, which might be a self-organizing process that switches uncoordinated neuronal activity to coupled oscillations, while high dimensionality and short delay correspond to EEG desynchronization that is plausibly the outcome of weakly coupled oscillations of neuronal assembly with out of phase synchrony [11]. As shown in our experimental observations (Figure 2 to 4), sleep stages of WF, LS and REM corresponding to the desynchronized EEG show high dimensionality and short delay, while sleep stage of DS corresponding to synchronized EEG indicates low dimensionality and long delay. Similarly, as shown in Figure 5, the EEG desynchronized stages of WF, LS and REM (showing highly exponential divergence pattern) can be characterized by the high value of the largest Lyapunov exponent, while the EEG synchronized stage of DS (showing low exponential divergence pattern) can be characterized by with low value of largest Lyapunov exponent. The observed finite values of the largest Lyapunov exponent for all cases (Figure 5) may indicate that trajectories of dissipative dynamical systems converge to a bounded subset of the phase space with increasing time [12, 13]. However, the mean values for those parameters in phase space transformation are not sufficient to discriminate wakefulness, light sleep and the REM, because those have the similar EEG signal patterns. Nevertheless, observed SEM difference between WF and REM can provide the cue to classify the WF and REM where they show similar physiological status.

## 5. CONCLUSIONS

In this paper, we analyze the EEG signals by means of chaotic methods including the time delay, the embedded dimension, the correlation dimension and the largest Lyapunov exponent over phase space representation. The embedded dimension, the correlation dimension and the largest Lyapunov exponent decrease as the sleep stage moves from the light sleep to deep sleep, while the time delay changes to opposite. However, they are not significant to catch up with light sleep stages (light sleep 1, and light sleep 2) and deep sleep stages (deep sleep 1 and deep 2). The SEMs of delay time, embedded dimension, correlation dimension and Lyapunov exponent in WF and LS1 show larger variability than that in REM, which can be helpful to discriminate WF and REM. The experimental results demonstrate the feasibility to functionally discriminate deep sleep and light sleep using EEG signals. The non-linear dynamics based on chaotic analysis can be useful to represent non-linear complex brain function associated with sleep. Further study will be performed to test more data samples to supplement individual variation and time-varying characteristics of the brain dynamics.

EEG signals are frequently generated by coupled oscillations of relevant neurons depending on the brain's status. However, the brain activity of oscillations sometimes occupies a relatively narrow time window and narrow spectral band depending on the brain's status.

Nevertheless, whether EEG signals is deterministic or chaotic is still controversial.

Analysis of complex dynamical systems such as non-linear dissipative system is gained much attraction.

There is a need to measure the degree of complexity and chaotic.

The correlation dimension can converge to the finite value for deterministic signal, but it can diverge for random signal, which can mostly be explained by estimating the correlation dimension and the Lyapunov exponent.

As the human goes toward deeper sleep, human physiological status becomes relatively stable, and many nerve cells are inactive, so the brain becomes a less complex system. However, whether the performance of chaotic analysis will overwhelm that of linear analysis methods or not is not clear. Further study will be performed to explain time varying non-linear brain dynamics.

The chaotic characteristics of the complexity and the exponential divergence are related with the correlation dimension, and the largest Lyapunov exponent, respectively.

## ACKNOWLEDGEMENTS

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